DATA ANALYSIS

Week 13: Additional predictors

logistics

- PS6 revisions due TODAY
- PS7 opt-out deadline Apr 23
- PS7 due Apr 30
- class participation:
    - Canvas discussion board posts due Apr 30
    - “practice” questions (10 multiple-choice/true-false) due Apr 24
- LAST DAY to submit any late work: May 13

Week 13: Additional Predictors

- Opt-out of Problem Sets (Deadline 3: After Midterm 2)
  Apr 23 | 1 pts
- Problem Set 7: First Attempt
  Apr 30 | 2.5 pts
- Problem Set 7: Second Attempt
  May 8 | 2.5 pts

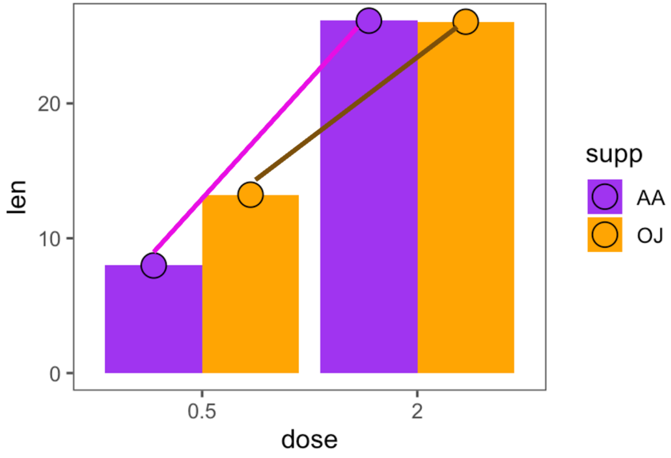
Class Participation

- Data Around Us!
  Apr 30 | 5 pts
- Meme Submission
  1 pts
- Student Practice Questions
  Apr 24 | 2.5 pts

12	F: April 12, 2024	**Exam (Midterm) 2**
13	W: April 17, 2024	W13: Additional Predictors
13	F: April 19, 2024	W13 continued…
14	T: April 23, 2024	**Problem Set Opt-out Deadline 3**
14	W: April 24, 2024	W14: Non-Independent/Miscellaneous Data
14	F: April 26, 2024	W14 continued…
15	T: April 30, 2024	**Problem Set 7 due**
15	W: May 1, 2024	W15: Odds and Ends
15	F: May 3, 2024	**Final Exam**
16	W: May 8, 2024	**Wrapping Up!**

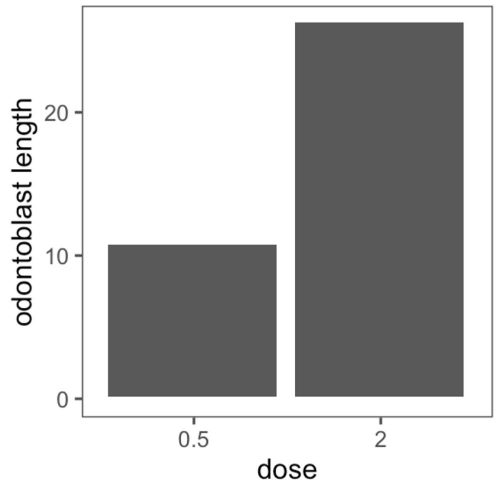
the tooth growth dataset

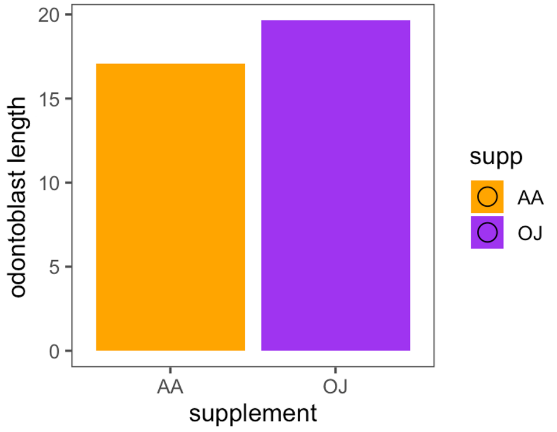
- this in-built R dataset contains the "length of odontoblasts (cells responsible for tooth growth) in 60 guinea pigs. each animal received one of three dose levels of vitamin C (0.5, 1, and 2 mg/day) by one of two delivery methods, orange juice or ascorbic acid"
- 2 (dose: 0.5 vs 1 mg) x 2 (supp: AA vs. OJ) design



building a factorial model

- we can start with three simple models
- grand mean model : toothGrowth ~ **grand mean**
- main effect 1: toothGrowth ~ **dose**
  - model = dose means
  - obtain $SS_{dose\_model} = SS_{total} - SS_{Y-\hat{Y}_{dose\_model}}$
- main effect 2: toothGrowth ~ **supp**
  - model = supplement means
  - obtain $SS_{supp\_model} = SS_{total} - SS_{Y-\hat{Y}_{supp\_model}}$





activity: compute the means

supplement	dose=0.5	dose=2
AA	7.98	26.14
OJ	13.23	26.06

difference

$AA_{0.5mg} – AA_{2mg}$ = -18.16

$OJ_{0.5mg} – OJ_{2mg}$ = -12.83

difference of differences = interaction

$(AA_{0.5mg} – AA_{2mg}) - (OJ_{0.5mg} – OJ_{2mg})$ = -5.33

AA_overall	17.06
OJ_overall	19.645
dose_0.5	10.605
dose_2	26.1

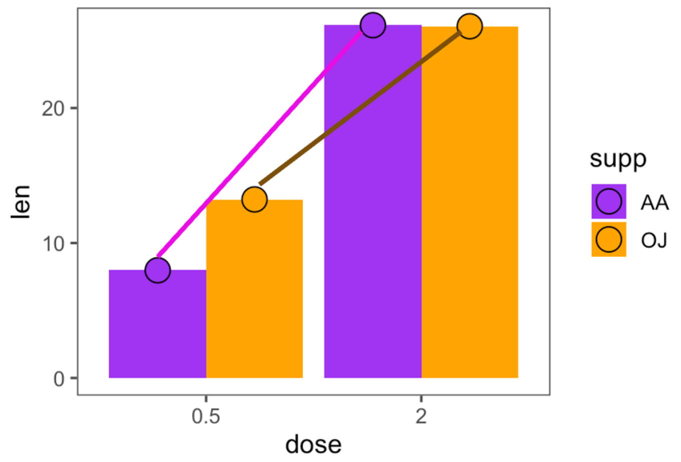
main effect of **supplement**

$M_{OJ} – M_{AA}$ = 2.585

main effect of **dose**

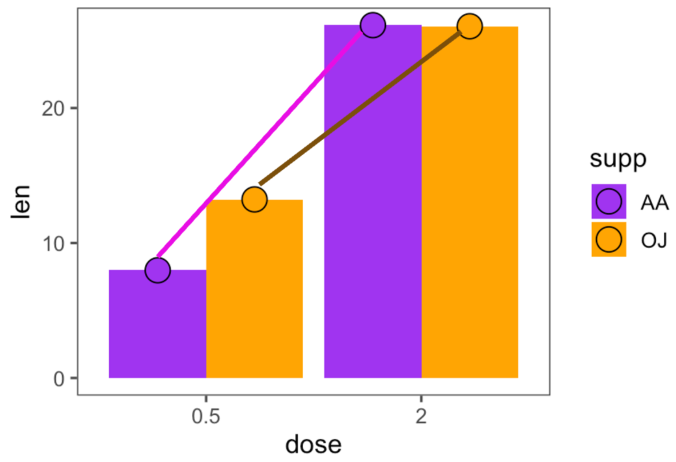
$M_{0.5mg} – M_{2mg}$ = 15.495

activity: build the models



- build the **grand mean** model
  - obtain SS_{total}
- build the **dose** model using dose means
  - obtain $SS_{dose_{model}}$
- build the **supplement** model using supplement means
  - obtain $SS_{supp_{model}}$

activity: build the models

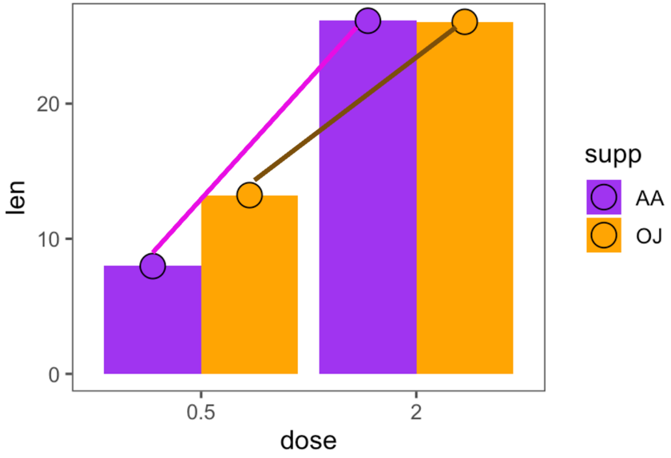


- build the **grand mean** model
  - obtain $SS_{total} = 3056.29975$
- build the **dose** model using dose means
  - obtain $SS_{dose_{model}} = 2400.95025$
- build the **supplement** model using supplement means
  - obtain $SS_{supp_{model}} = 66.82225$

SStotal	3056.29975

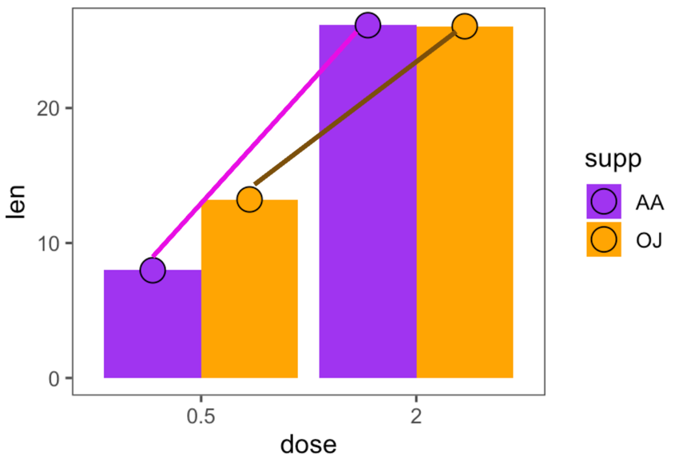
	SS
supplement_model	66.82225
dose_model	2400.95025

building a complex model



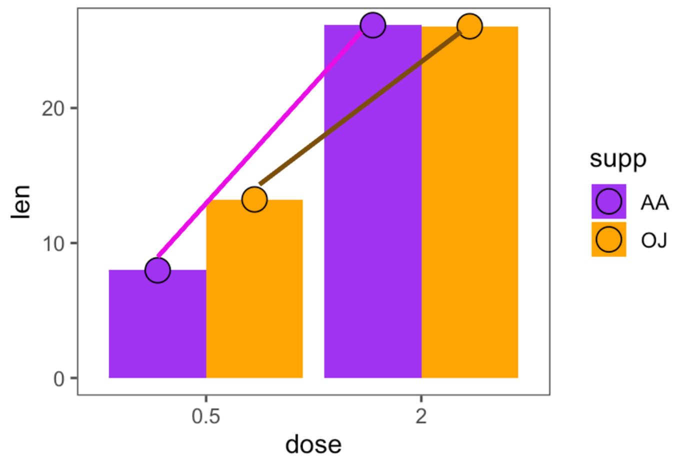
- next, we fit our more complex model
- interaction model: toothGrowth ~ **dose + supp** + (**dose**)(**supp**)
  - substitutes each value with the respective sub-mean of the factorial design
  - obtain $SS_{full\_model} = SS_{total} - SS_{Y-\hat{Y}_{full\_model}} = SS_{total} - SS_{error}$
- how much variance is explained by the interaction ($SS_{interaction}$)?
  - $SS_{interaction} = SS_{full\_model} - SS_{dose_{model}} - SS_{supp_{model}}$
- the interaction represents the part of the "full model" that is not explained by the simple models of only dose and only supplement

activity: build full model



- build full model using <u>all</u> sub-group means
  - $SS_{error} = ??$ (the error left over from the full model)
    - also called $SS_{residuals}$
  - $SS_{full\_model} = SS_{total} - SS_{error} = ??$
  - $SS_{interaction} = SS_{full\_model} - SS_{dose_{model}} - SS_{supp_{model}}$
  - $SS_{interaction} = ??$

activity: build full model



- build full model using <u>all</u> sub-group means
  - $SS_{error} = 517.505$ (the error left over from the full model)
    - also called $SS_{residuals}$
  - $SS_{full\_model} = SS_{total} - SS_{error} = 2538.79475$
  - $SS_{interaction} = SS_{full\_model} - SS_{dose_{model}} - SS_{supp_{model}}$
  - $SS_{interaction} = 71.02225$

	SS
supplement_model	66.82225
dose_model	2400.95025
interaction	71.02225
residuals	517.505
SStotal	3056.29975

NHST for factorial ANOVA

step 1: build grand mean model

(1) "summarizing" data using a single grand mean (ignoring all group labels)

(2) compute SS_{total}

step 2: build factor 1 model

(1) "summarize" data using the means for levels from the **first** independent variable

(2) compute SS_{model1} and SS_{error1}

step 3: build factor 2 model

(1) "summarize" data using the means for levels from the **second** independent variable

(2) compute SS_{model2} and SS_{error2}

step 4: build full model

(1) "summarize" data using the means for the **full** 2x2 design (i.e., each of the 4 means)

(2) compute $SS_{fullmodel}$ and SS_{error}

(3) compute $SS_{interaction}$

step 5: build F table & conduct ALL F tests!

(1) create F table

(2) find $F_{critical}$

(3) compute $F_{observed} = \frac{MS_{model}}{MS_{error}}$

(4) find p-value for F-score

(4) decide!

testing significance (F-test)

- we conduct individual F-tests for each type of possible effect using the remaining error ($SS_{residual}$) from the full model

$$F(df_1, df_2) = \frac{MS_{model}}{MS_{error}} = \frac{SS_{model}/df_{model}}{SS_{error}/df_{error}}$$

- degrees of freedom
  - $df_{1i} = k_i - 1$
  - $df_{interaction} = product\ of\ all\ df_{1i}$
  - $df_2 = \mathrm{n}\ - \text{product of } k_i$

df for toothGrowth dataset

n	k	term	df	
40				

df for toothGrowth dataset

n	k	term	df	
40	2 (AA vs. OJ)			
	2 (0.5 mg vs 2 mg)			

df for toothGrowth dataset

n	k	term	df	
40	2 (AA vs. OJ)	supplement		
	2 (0.5 mg vs 2 mg)	dose		
		interaction		
		residual		

df for toothGrowth dataset

n	k	term	df	
40	2 (AA vs. OJ)	supplement	2-1 = 1	
	2 (0.5 mg vs 2 mg)	dose	2-1 = 1	
		interaction	1 x 1 = 1	
		residual	40 – (2*2) = 36	error or within

practice question

- For an experiment involving 2 levels of factor A and 3 levels of factor B with a sample of n = 5 in each treatment condition, what is the value for df_{within}?
  - 20
  - 24
  - 29
  - 30

practice question

- The results of a two-factor analysis of variance produce df = 2, 36 for the F-ratio for factor A and df = 2, 36 for the F-ratio for factor B. What are the df values for the AxB interaction?
  - 1, 36
  - 2, 36
  - 3, 36
  - 4, 36

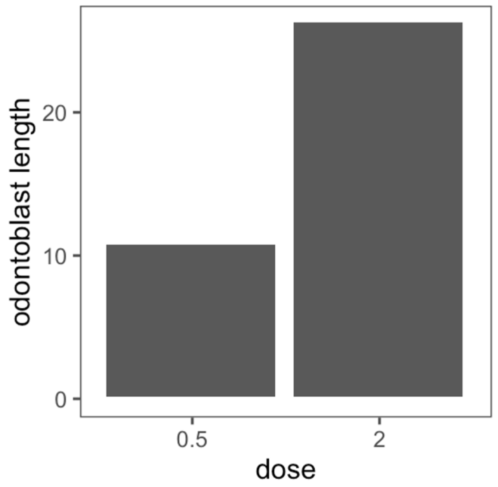
testing significance (F-test)

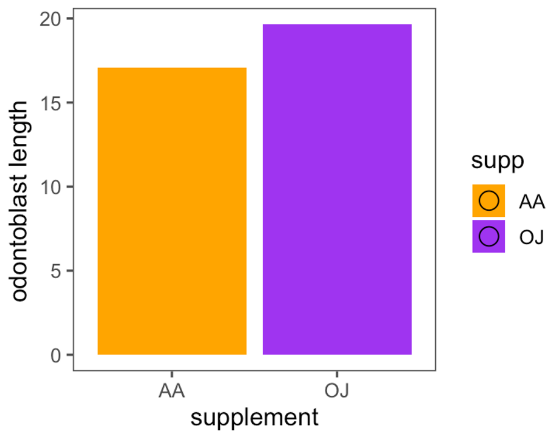
k		SS	df	MS	F_observed	F_critical	check	p_value
2	**supplement_model**	66.82225	1	66.82225	4.648459435	4.1132	TRUE	0.0378
2	**dose_model**	2400.95025	1	2400.95025	167.0210124	4.1132	TRUE	less than 0.0001
	interaction	71.02225	1	71.02225	4.940630525	4.1132	TRUE	0.0326
	residuals	517.505	36	14.37513889				
	SStotal	3056.29975						

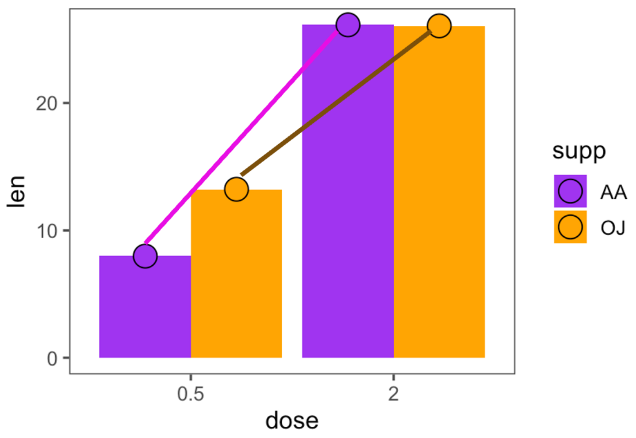
Sheets solution
video tutorial

post-hoc tests

- once the "overall" F-tests show that substantial variation is explained by some combination of independent variables, we can dive in and explore specific effects
- sometimes, researchers have specific hypotheses about main effects and/or the interaction(s)
- these hypotheses can be tested using pairwise t-tests/one-way ANOVAs, but must be corrected for multiple comparisons







continuous IVs

- the same framework in general holds for interval/ratio-level independent variables
  - *multiple regression*: $Y = b_1X_1 + b_2X_2 + \ldots + a + \text{error}$
- here, the coefficients represent the change in Y as a function of the specific independent variable (X_i) when "controlling for" the effect of other variables
- just as the linear correlation is structurally equivalent to the slope of a line, *partial* correlations are structurally equivalent to the coefficients from a multiple regression
- interactions are products of the two variables (similar to covariance!)

multiple regression formula

- fitting a (multiple) regression model in Sheets / Excel
- LINEST(Y, range of X columns/predictors, TRUE, FALSE)
- interpreting coefficients of a multiple regression helps you understand the impact of specific variables
- [Sheets example](#) for mtcars

H24 fx =LINEST(B2:B33,C2:E33,TRUE,FALSE)

	A	B	C	D	E
1	:ar	mpg (Y)	hp (X1)	wt (X2)	product (X3)
2	Mazda RX4	21	110	2.62	288.2
3	Mazda RX4 Wag	21	110	2.875	316.25
4	Datsun 710	22.8	93	2.32	215.76
5	Hornet 4 Drive	21.4	110	3.215	353.65
6	lornet Sportabout	18.7	175	3.44	602

d (hp)(wt)	c (wt)	b (hp)	a
0.02784814832	-8.216624297	-0.120102091	49.80842343

next time

- **before** class
  - *watch:* Hypothesis Testing (Factorial ANOVA) [33 min]
  - *explore:* Problem Set 7!
  - *post*: Data Around Us OR practice questions (class participation)
- **during** class
  - miscellaneous data (repeated measures + non-parametric)

optional: building a complex model

- what is our model's equation?
  - toothGrowth ~ a + b (dose) + c (supp) + d (dose) (supplement)
  - simple coefficients signify main effects (b and c)
  - product coefficients signify interactions
  - "intercept" (a) signifies the mean of toothGrowth when all other coefficients = 0
  - NOTE: this is no longer a line!
- what are the values of a, b, c, and d?
  - nominal independent variables are converted to 0s and 1s ("dummy codes")
  - intercept (a): dose and supp are both 0, i.e., predicted mean toothGrowth in the $AA_{0.5mg}$ group
  - b: dose = 1, supp = 0, i.e., change in toothGrowth from $AA_{0.5mg}$ to AA_{2mg}
  - c: supp = 1, dose = 0, i.e., change in toothGrowth from $AA_{0.5mg}$ to $OJ_{0.5mg}$
  - d: supp = 1, dose = 1, i.e., difference of differences, i.e., $(OJ_{0.5mg} - OJ_{2mg}) - (AA_{0.5mg} - AA_{2mg})$
- this is called **dummy coding** or setting up **contrasts** in your model

	0	1
dose	0.5mg	2mg
supp	AA	OJ

optional: building a complex model

- "dummy coding" each factor
- then using LINEST
- provides you a linear model's equation
- see last table of Sheets solution!

```
=LINEST(B2:B41,F2:H41,TRUE,FALSE)
```

SUPP_DUMMY	DOSE_DUMMY	SUPP*DOSE		interaction	dose_0.5	supp_OJ	INTERCEPT
0	0	0		-5.33	18.16	5.25	7.98
0	0	0				AA-OJ 0.5	AA_0.5
0	0	0					
0	0	0					