DATA ANALYSIS

Week 14: Non-parametric tests

logistics

- extra credit surveys + memer contest will be live today (due next Monday)
- extra credit research assignment is due May 15
- no late submissions will be accepted or graded past 1 pm May 15

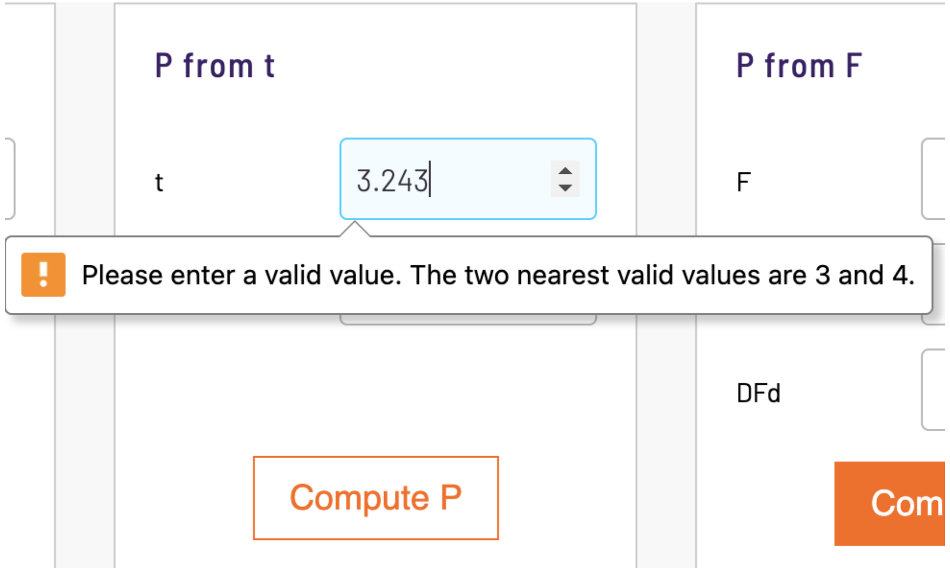
15	M: April 28, 2025	**PS5+ PS6 revision due**
15	T: April 29, 2025	W15: Miscellaneous Data
15	Th: May 1, 2025	W15 continued…
16	M: May 5, 2025	**PS7 due**
16	T: May 6, 2025	W16: Last Class / Final Exam review
17	Th: May 15, 2025	**PS7 revision + Computational Exam Due by 1.30 pm**
17	Th: May 15, 2025	**Conceptual Exam (1.30-3 pm, VAC South)**

lingering questions

- For problem sets, is 75% completion only for the first attempt? (will we need to finish 100% of the problems for the revision?)
- Can we review what it means for an interaction to be "difference of differences"

alternative p-value calculators

- https://365datascience.com/calculators/p-value-calculator/
- https://www.socscistatistics.com/pvalues/
- https://www.omnicalculator.com/statistics/p-value



assumptions of tests thus far

- interval/ratio DV
- normality of dependent variable (for large samples, this can be relaxed a bit)
  - for ANOVAs, normality within each level of IV
- fully independent/unpaired or fully dependent/paired data
- homogeneity of variance
  - for repeated measures ANOVA, sphericity also applies

parametric vs. non-parametric tests

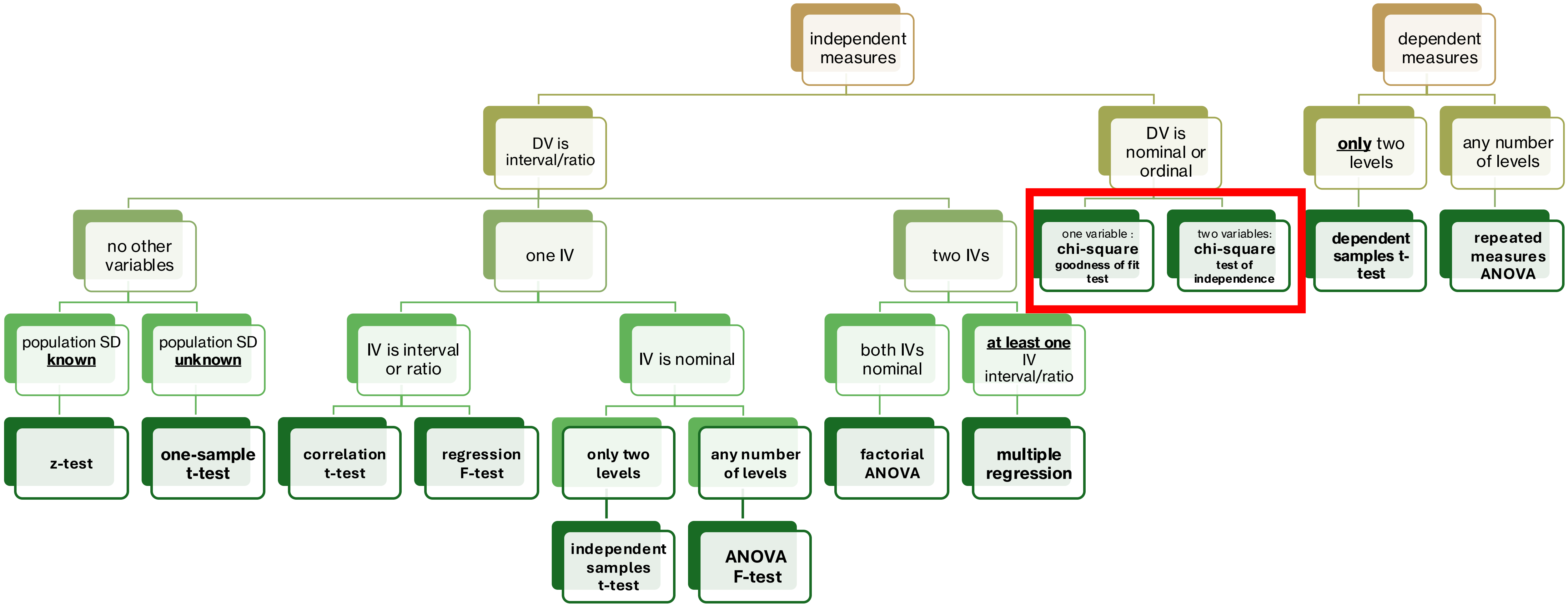
parametric tests

- interval/ratio DVs
- involve estimating parameters
- assumptions about the underlying sampling distribution
- if assumptions are violated, these tests may not be appropriate

non-parametric tests

- assume no underlying distributions (“distribution-free”)
- typically used for nominal/ordinal DVs that yield counts
- no assumptions about underlying population
- most parametric tests have a non-parametric alternative

hypothesis testing flowchart

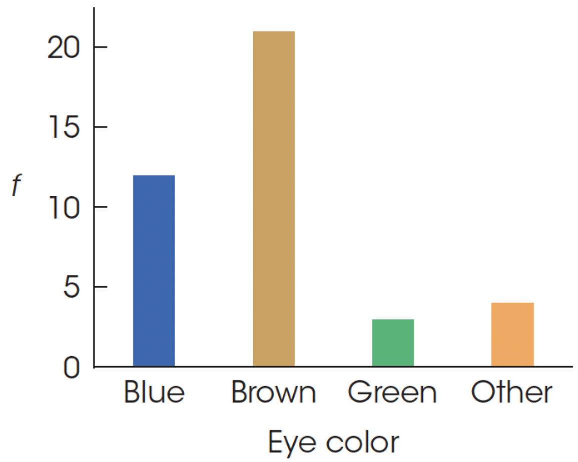


chi-square tests

- chi-square goodness of fit test
  - one nominal/ordinal variable
  - asks whether <u>observed</u> distribution of responses matches <u>expected</u> distribution
- chi-square test of independence
  - two nominal/ordinal variables
  - asks whether observed distribution of responses on one variable depends on responses on other variable

example: eye color

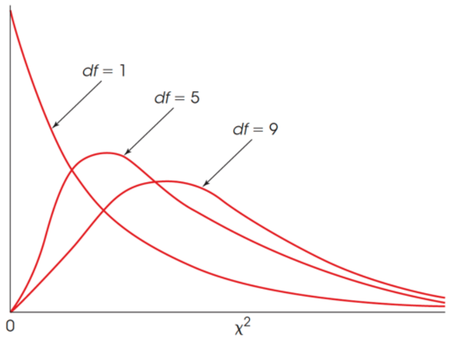
- eye color counts for 40 students
- can be represented in a bar graph or frequency distribution table
- counts typically converted to a table
- observed values/counts are then compared to expected values/counts via a ratio
- asking: how extreme are the differences between what is expected and what is observed?



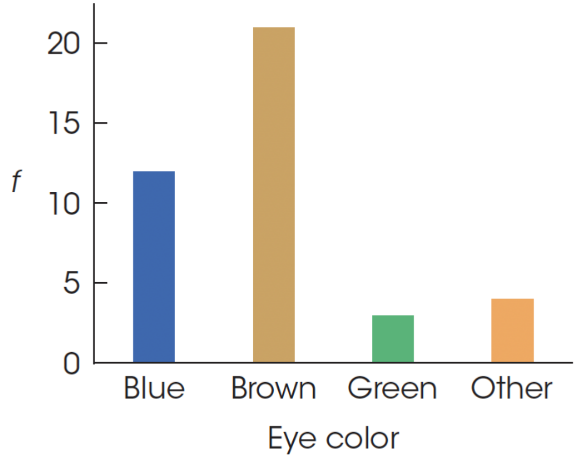
Eye color (X)	f
Blue	12
Brown	21
Green	3
Other	4

	blue	brown	green	other
observed (f_o)	12	21	3	4
expected (f_e)				

chi-square goodness of fit test



- $\chi^2 = \sum \frac{(f_o - f_e)^2}{f_e}$
- the "expected" frequencies form the null hypothesis (H_0)
  - equal preference (all counts equal)
  - known population (specific distribution)
- observed χ^2 statistic is then compared to the expected distribution for a set degrees of freedom based on number of categories C
  - $df = C - 1$



Eye color (X)	f
Blue	12
Brown	21
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Other	4

	blue	brown	green	other
observed (f_o)	12	21	3	4
expected (f_e)	10	10	10	10

$$f_e = \frac{N}{C} \quad for\ equal\ preference$$

NHST for chi-square goodness of fit test

step 1: state the hypotheses

$H_0: equal\ preference\ OR\ known\ distribution$

$H_1: distribution\ does\ not\ match\ expected\ distribution$

step 2: set criteria for decision

$\alpha = .05$

find $\chi^2{}_{critical}$ based on **<u>right tailed</u>** test and degrees of freedom

$df = C - 1$

step 3: collect data

(1) find observed frequencies f_o

(2) find expected frequencies f_e

$f_e = \frac{N}{C}\ for\ equal\ preference$

$f_e = N\,(p_k)\ for\ expected\ proportions$

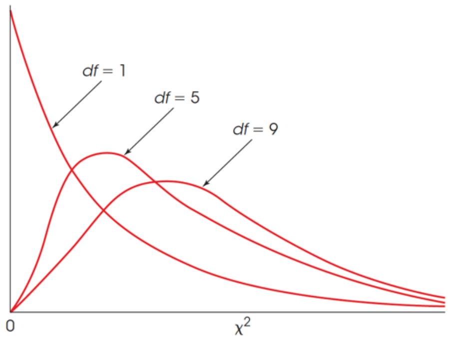
(3) compute $\chi^2{}_{observed} = \sum \frac{(f_o - f_e)^2}{f_e}$

(4) find p-value for $\chi^2{}_{observed}$

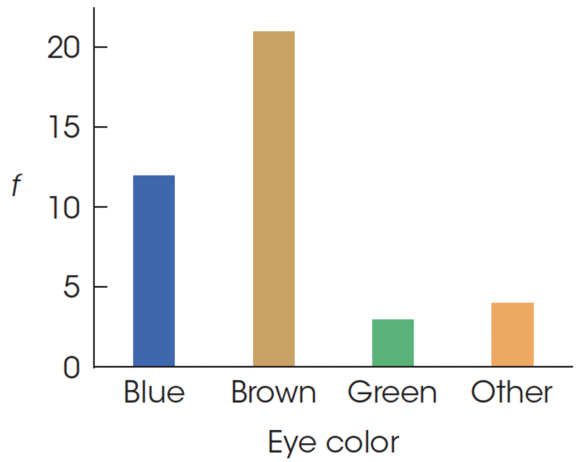
step 4: make a decision!

check whether $\chi^2{}_{observed}$ is beyond $\chi^2{}_{critical}$ and p-value < .05. if so, reject null hypothesis!

chi-square goodness of fit test



- $C = 4$
- $df = C - 1 = 3$
- $\chi^2{}_{critical}\ (3) = 7.8147$
- $\chi^2{}_{observed} = \sum \frac{(f_o - f_e)^2}{f_e} = 21$
- p-value < .0001
- APA reporting: A significant difference was observed in eye color distributions, $\chi^2\ (3, n = 40) = 21, p < \ .0001$



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Blue	12
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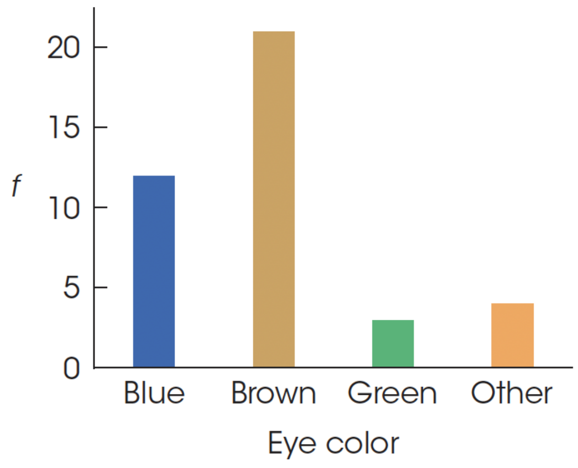
	blue	brown	green	other
observed (f_o)	12	21	3	4
expected (f_e)	10	10	10	10

known distribution

Eye Color	U.S. Population	World Population
Gray and other	Less than 1%	Less than 1%
Green	9%	2%
Hazel/amber	18%	10%
Blue	27%	8% to 10%
Brown	45%	55% to 79%

- has eye color significantly changed in the US population since 2000?
- our hypothesis is no longer about equal preference, but instead about a known population distribution
- $f_e = N\,(p_k)\ for\ expected\ proportions$
- $f_e\,(blue) = 40\,(.27) = 10.8$
- $f_e\,(other) = 40\,(.18 + .01) = 7.6$



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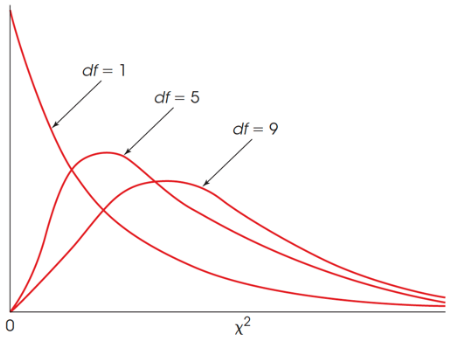
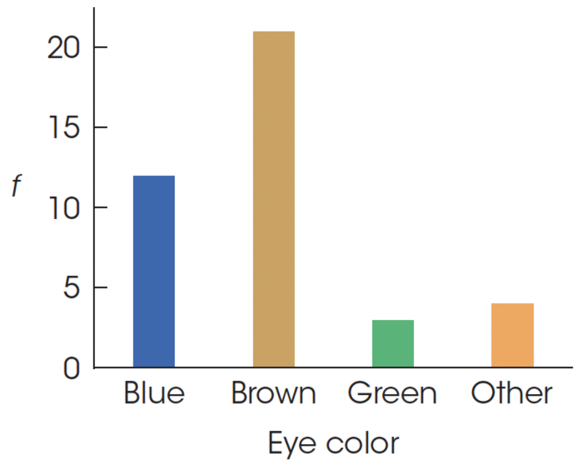
observed (f_o)	12	21	3	4
expected (f_e)	10.8	18	3.6	7.6

$$f_e = N\,(p_k)\ for\ expected\ proportions$$

chi-square goodness of fit test



- $C = 4$
- $df = C - 1 = 3$
- $\chi^2{}_{critical}\ (3) = 7.8147$
- $\chi^2{}_{observed} = \sum \frac{(f_o - f_e)^2}{f_e} = 2.438$
- p-value = 0.4865
- APA reporting: Eye color distributions have not significantly changed since 2000, $\chi^2\ (3, n = 40) = 2.43, p = \ .49$



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W14 Activity 1

- data
- conduct a chi-square test

Data from the Motor Vehicle Department indicate that 80% of all licensed drivers are older than age 25.

a. In a sample of $n = 50$ people who recently received speeding tickets, 33 were older than 25 years and the other 17 were age 25 or younger. Is the age distribution for this sample significantly different from the distribution for the population of licensed drivers? Use $\alpha = .05$.

chi-square test for independence

- is parent-allowed alcohol use related to how many alcohol-related problems are experienced?
- typically, this is a situation where there is no clear IV/DV but a relationship needs to be tested
- this is a non-parametric version of a correlation test
- note that variables are no longer interval/ratio: these are COUNTS

OBSERVED frequencies		experienced alcohol-related problems		
		yes	no	
parents allowed alcohol use	allowed	71	9	
	not allowed	89	31	

chi-square test for independence

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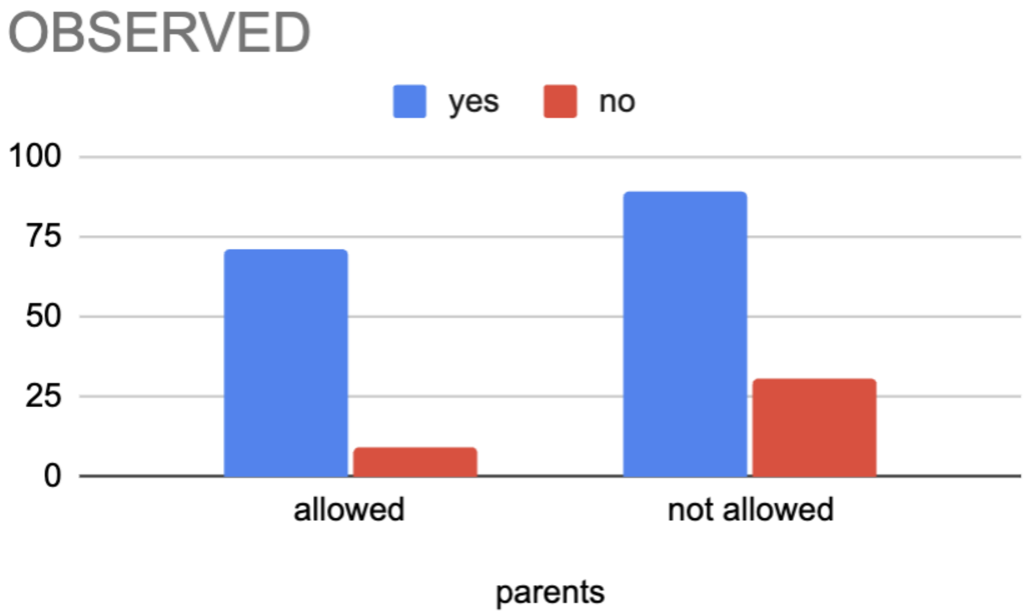

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OBSERVED frequencies		experienced alcohol-related problems		
		yes	no	
parents allowed alcohol use	allowed	71	9	
	not allowed	89	31	

chi-square test for independence

- we first count up the totals row and column-wise to get how many people were sampled and how many were in each level

OBSERVED frequencies		experienced alcohol-related problems		
		yes	no	total
parents allowed alcohol use	allowed	71	9	80
	not allowed	89	31	120
	total	160	40	N = 200

expected frequencies

- what proportion of students experienced problems?
  - 160 / 200 = .80
- if problems experienced **are not related to whether parents allowed alcohol use or not**, then 80% of the students should experience problems and 20% shouldn't
  - expected (allowed-yes) = .80 * 80 = 64
  - expected (allowed-no) = .20*80 = 16

EXPECTED frequencies		experienced alcohol-related problems		
		yes	no	total
parents allowed alcohol use	allowed			80
	not allowed			120
	total	160	40	N = 200
		.80	.20	

expected frequencies

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- if problems experienced **are not related to whether parents allowed alcohol use or not**, then 80% of the students should experience problems and 20% shouldn't
  - expected (allowed-yes) = .80 * 80 = 64
  - expected (allowed-no) = .20*80 = 16

EXPECTED frequencies		experienced alcohol-related problems		
		yes	no	total
parents allowed alcohol use	allowed	64	16	80
	not allowed			120
	total	160	40	N = 200
		.80	.20	

expected frequencies

- what proportion of students experienced problems?
  - 160 / 200 = .80
- if problems experienced **are not related to whether parents allowed alcohol use or not**, then 80% of the students should experience problems and 20% shouldn't
  - expected (not allowed-yes) = .80 * 120 = 96
  - expected (not allowed-no) = .20*120 = 24

EXPECTED frequencies		experienced alcohol-related problems		
		yes	no	total
parents allowed alcohol use	allowed	64	16	80
	not allowed			120
	total	160	40	N = 200
		.80	.20	

expected frequencies

- what proportion of students did NOT experience problems?
  - 40 / 200 = .20
- if problems experienced **are not related to whether parents allowed alcohol use or not**, then 80% of the students should experience problems and 20% shouldn't
  - expected (not allowed-yes) = .80 * 120 = 96
  - expected (not allowed-no) = .20*120 = 24

EXPECTED frequencies		experienced alcohol-related problems		
		yes	no	total
parents allowed alcohol use	allowed	64	16	80
	not allowed	96	24	120
	total	160	40	N = 200
		.80	.20	

NHST for chi-square test of independence

step 1: state the hypotheses

$H_0: no\ relationship\ between\ variables$

$H_1: there\ is\ a\ relationship\ between\ variables$

step 2: set criteria for decision

$\alpha = .05$

find $\chi^2_{critical}$ based on **right tailed** test and degrees of freedom

$df = (R-1)(C-1)$

step 3: collect data

(1) find observed frequencies f_o

(2) find expected frequencies f_e based on proportions

(3) compute $\chi^2_{observed} = \sum \frac{(f_o - f_e)^2}{f_e}$

(3) find p-value for $\chi^2_{observed}$

step 4: make a decision!

check whether $\chi^2_{observed}$ is beyond $\chi^2_{critical}$ and p-value < .05. if so, reject null hypothesis!

chi-square test

- $df = (R-1)(C-1)$
- $df = (2-1)(2-1) = 1$
- $\chi^2{}_{critical}\ (1) = 3.84$
- $\chi^2{}_{observed} = \sum \frac{(f_o - f_e)^2}{f_e} = 6.38$
- p-value = 0.0115

OBSERVED frequencies		experienced alcohol-related problems		
		yes	no	total
parents allowed alcohol use	allowed	71	9	80
	not allowed	89	31	120
	total	160	40	N = 200

EXPECTED frequencies		experienced alcohol-related problems		
		yes	no	
parents allowed alcohol use	allowed	64	16	80
	not allowed	96	24	120
		160	40	N = 200

W14 Activity 2

- data

In a classic study, Loftus and Palmer (1974) investigated the relationship between memory for eyewitnesses and the questions they are asked. In the study, participants watched a film of an automobile accident and then were questioned about the accident. One group was asked how fast the cars were going when they "smashed into" each other. A second group was asked about the speed when the cars "hit" each other, and a third group was not asked any question about the speed of the cars. A week later, the participants returned to answer additional questions about the accident, including whether they recalled seeing any broken glass. Although there was no broken glass in the film, several students claimed to remember seeing it. The following table shows the frequency distribution of responses for each group.

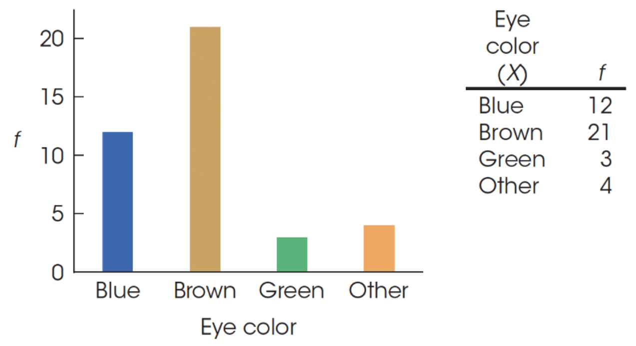
Response to the Question Did You See Any Broken Glass?

	Verb Used to Ask About the Speed	Response About Broken Glass: Yes	Response About Broken Glass: No
Verb Used to Ask About the Speed	Smashed into	16	34
	Hit	7	43
	Control (Not Asked)	6	44

a. Does the proportion of participants who claim to remember broken glass differ significantly from group to group? Test with $\alpha = .05$.

chi-square test: assumptions

- independence of observations (between-subject measurements)
- expected frequencies in each cell > 5
- typically categories are merged if counts are low



	blue	brown	green	other
observed (f_o)	12	21	3	4
expected (f_e)	10.8	18	3.6	7.6

$$f_e = N\ (p_k)\ for\ expected\ proportions$$

next time

- bootstrapping

Before Thursday

- Watch Bootstrapping Statistics
- Work on: Reviewing all tests.

Here are the to-do's for this week:

- Submit Problem Set 7
- Submit any lingering questions here!
- Final extra credit opportunities:
  - Final class survey
  - Vote in the Memer Contest
  - Complete Attitudes Survey